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Name Israa, ID 900097435

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Quiz I

Ayman Badawi

QUESTION 1. (6 points) Show that the following repeated decimals numbers are indeed rational numbers

(i) $x = 3.2\bar{8}\bar{9}$

$100x = 328.\overline{989}$

$100x - x = 325.7$

$99x = \frac{3257}{10}$

$\boxed{x = \frac{3257}{990}}$ ✓ ③

(ii) $x = 23.8872\bar{5}$

$10x = 238.872\bar{5}$

$= 238.8725\bar{5}$

$10x - x = 214.9853$

$\rightarrow 9x = \frac{2149853}{10000}$

$\boxed{x = \frac{2149853}{90000}}$ ✓ ③

QUESTION 2. (6 points)

(i) Solve for x. $6x = 4$ over planet Z_8.

gcd(6,8) = 2

is 2|4? yes

∴ there exists 2 solutions $\rightarrow$

$x_1 = 2$

$x_2 = 2 + \frac{8}{2} = 2 + 4 = 6$

$\boxed{\text{solution set: } \{2, 6\}}$ ✓ ③

(ii) Solve for x, $3x = 6$ over planet Z_9.

gcd(3,9) = 3

is 3|6? yes

∴ there exists 3 solutions $\rightarrow$

$x_1 = 2$

$x_2 = 2 + \frac{9}{3} = 2 + 3 = 5$

$x_3 = 5 + 3 = 8$

$\boxed{\text{solution set: } \{2, 5, 8\}}$ ✓ ③

QUESTION 3. (8 points) Find the smallest positive integer x such that $x \pmod 9 = 7$ and $x \pmod 5 = 3$

$m_1 = 9$, $m_2 = 5$

$r_1 = 7$, $r_2 = 3$

gcd(9,5) = 1

∴ CRT confirms that there exists a solution ✓

$m = 9 \times 5 = 45$

$n_1 = \frac{45}{9} = 5$

$n_2 = \frac{45}{5} = 9$

⑤

n_1^{-1} in $Z_{m_1} \rightarrow (5)^{-1}$ in $Z_9 = 2$

n_2^{-1} in $Z_{m_2} \rightarrow (9)^{-1}$ in $Z_5 = 4$

$x = [5 \times 2 \times 7 + 9 \times 4 \times 3] (\text{mod } 45)$

$= [178] \text{ mod } 45$

$\boxed{x = 43}$ ✓

Name Israa, ID 900097435

$\frac{20}{20}$

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Quiz II

Ayman Badawi

QUESTION 1. (6 points)

(i) $(677)_8 \times (7)_8$.

$$\begin{array}{r} {\scriptstyle 6\ 6} \\ (677)_8 \\ (7)_8 \\ \hline (6071)_8 \end{array} = \boxed{(6071)_8}$$

✓ $2/2$

(ii) $(636)_7 - (242)_7$

$$\begin{array}{r} {\scriptstyle 5\ 10} \\ (636)_7 \\ (242)_7 \\ \hline (364)_7 \end{array} = \boxed{(364)_7}$$

✓ $2/2$

(iii) $(878)_9 + (888)_9$

$$\begin{array}{r} {\scriptstyle 1\ 1} \\ (878)_9 \\ + (888)_9 \\ \hline (1877)_9 \end{array} = \boxed{(1877)_9}$$

✓ $2/2$

QUESTION 2. As explained in the class find $gcd(133, 28)$.

$$\begin{array}{r} 4 \\ 28 \overline{)133} \\ -112 \\ \hline 21 \end{array} \nearrow \begin{array}{r} 1 \\ 21 \overline{)28} \\ -21 \\ \hline 7 \end{array} \nearrow \begin{array}{r} 3 \\ ⑦ \overline{)21} \\ -21 \\ \hline \underline{\underline{0}} \end{array} \text{ stop.} \implies \boxed{gcd(133, 28) = 7}$$

✓ $3/3$

QUESTION 3. (5 points)

(i) Let $n = 39 \times 26$. Find $\phi(n)$.

$n = 3 \times 13 \times 13 \times 2$

$n = 3 \times 13^2 \times 2$ $\rightarrow$ $\phi(n) = 3^0(2) \times 13^1(12) \times 2^0(1)$

$\boxed{= 312}$ ✓

$2.5/2.5$

(ii) Let $n = 7^4 \times 11^2 \times 3^3$. Find $\phi(n)$.

$\phi(n) = 7^3(6) \times 11^1(10) \times 3^2(2)$

$\boxed{= 4074840}$ ✓

$2.5/2.5$

QUESTION 4. (6 points)

(a) Find 3^{80028} $(mod\ 25)$

$gcd(3, 25) = 1$

$\therefore$ Euler-Fermat's can be used

$25 = 5^2$

$\phi(25) = 5^1(4) = 20$ $\rightarrow$ $80028 \pmod{20} = 8$

$3^{80028} \pmod{25} = 3^8 \pmod{25}$

$\boxed{= 11}$

✓ $3/3$

(b) Find 7^{88026} $(mod\ 23)$

$gcd(7, 23) = 1$

$\therefore$ Euler-Fermat's can be used

$\phi(23) = 23^0(22) = 22$

$88026 \pmod{22} = 4$ $\rightarrow$ $7^{88026} \pmod{23} = 7^4 \pmod{23}$

$\boxed{= 9}$

✓ $3/3$

Name PRANAV VIVEK, ID B00090976

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Quiz III

Ayman Badawi

QUESTION 1. (1)(**4 points**) Let n be an odd integer. Prove directly that $3n + 7$ is an even integer, i.e., show that $3n + 7 = 2m$ for some integer m.

$n \rightarrow$ odd $\Rightarrow n = 2m+1$ for some integer $m \in \mathbb{Z}$

$3n+7 = 3(2m+1)+7 = 6m+3+7 = \overset{\text{even}}{\boxed{6m}+\boxed{10}} = 2(\underbrace{3m+5}_{\text{integer}})$ let $k = 3m+5$

$3n+7 = \text{~~6m+10~~} = 2k \rightarrow$ even. proved. ✓ 4/

(2)(**6 points**) Let n be an integer. Prove directly that $n^2 + 3n$ is an even integer. [Hint: Consider 2 cases. First case, assume n is even. Second case assume n is odd]

(i) n is even

$n = 2k$ for some integer $k \in \mathbb{Z}$

$n^2+3n = (2k)^2 + 3(2k) = 4k^2+6k = 2(\underbrace{2k^2+3k}_{\text{integer}})$

Let $m = 2k^2+3k$ $\therefore n^2+3n = 2m \rightarrow$ even ✓ 6/

(ii) n is odd

$n = 2k+1$ for some integer $k \in \mathbb{Z}$

$n^2+3n = (2k+1)^2 + 3(2k+1) = (2k)^2 + 1 + 2(2k) + 6k+3 = 4k^2+4k+6k+3+1 = 4k^2+10k+4$

$\Rightarrow 2(2k^2+5k+2)$ Let $m = 2k^2+5k+2$ $\therefore n^2+3n = 2m \rightarrow$ even ✓

Thus n^2+3n is an even integer for some integer $n \in \mathbb{Z}$. proved

(3) **5 points**) Let n be an irrational number and m be a rational number. Prove by contradiction that $n + m$ is irrational.

Deny that $n+m$ is irrational, $\therefore n+m = \frac{a}{b}$ 5/

$n = \overset{\text{rational}}{\boxed{\frac{a}{b}}} - \overset{\text{rational}}{\boxed{m}} \rightarrow$ ~~irrational~~ rational, a contradiction

$\therefore$ The denial is invalid, hence $x+y$ is irrational ✓

(4) (**5 points**) Find $LCM[24, 84]$

$\gcd(24,84) = 12$ 5/

$LCM(24,84) = \frac{24 \times 84}{\gcd(24,84)} = \frac{2016}{12} = 168$ ✓

$$24\overline{)84}^{\,3} \quad 72 \quad 12$$

$$12\overline{)24}^{\,02} \quad 24 \quad 0$$

Name Rimsha Razook, ID 96743

20/20.

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Quiz IV

Ayman Badawi

QUESTION 1. (1)(**8 points**) Use the 4th method and prove that $\sqrt{26}$ is irrational.

Deny $\sqrt{26}$ is rational

$\sqrt{26} = \frac{a}{b}$, a, b are integers, $b \neq 0$

$26 = \frac{a^2}{b^2}$ $\rightarrow a^2 =$ even, $\rightarrow b^2 =$ odd

$a = 2n$ for some integer n

$b = 2m+1$ for some integer m

$$\frac{26}{1} = \frac{(2n)^2}{(2m+1)^2} = \frac{4n^2}{4m^2+4m+1}$$

$$26 \cdot 4m^2 + 26 \cdot 4m + 26 = 4n^2$$

$$\underbrace{26m^2 + 26\overset{4}{m}}_{\text{irrational}} + \underbrace{\frac{26}{4}}_{\rightarrow \text{irrational}} = \cancel{8}\, n^2 \rightarrow \text{rational}$$

irrational cannot be equal to rational

∴ by contradiction $\sqrt{26}$ is irrational

✓

(2)(**4 points**) Use (1) above, and convince me that $\sqrt{2} + \sqrt{13}$ is irrational.

$\sqrt{2} + \sqrt{13} = \frac{a}{b}$, $a, b \in Z$, $b \neq 0$

$(\sqrt{2} + \sqrt{13})^2 = \frac{a^2}{b^2} = 2 + 2\sqrt{26} + 13$

$$(\sqrt{2}+\sqrt{13})(\sqrt{2}+\sqrt{13})$$
$$= 2 + \sqrt{2}\sqrt{13} + \sqrt{2}\sqrt{13} + 13$$
$$= 2 + 2\sqrt{26} + 13$$

$$\underset{\downarrow \text{ irrational}}{\sqrt{26}} = \underbrace{\frac{a^2}{2b^2} - \frac{15}{2}}_{\text{rational}} \rightarrow \text{not possible}$$

∴ by contradiction, $\sqrt{2} + \sqrt{13}$ is also irrational

✓

(3) **8 points**) Use math induction and prove that $14 \mid (3^{6n} - 1)$, for every $n \geq 1$.

1) $n = 1$
$14 \mid (3^6 - 1) = 14 \mid 728$ ✓

2) $14 \mid (3^{6n} - 1)$ is true for some n, $n \geq 1$

3) $14 \mid 3^{6(n+1)} - 1 = 14 \mid 3^{6n+6} - 1 = 14 \mid 3^{6n} \cdot 3^6 - 1$

$3^6 \cdot (3^{6n} - 1 + 1) - 1 = 3^6 (3^{6n} - 1) + 3^6 - 1$

↳ multiple so doesn't affect

$14 \mid 3^6 - 1$ by ①

$14 \mid 3^{6n} - 1$ by ②

∴ $14 \mid (3^{6n} - 1)$ for every $n \geq 1$

✓

Name: Mohamed Alawadhi ID: 600094286

DATE

20/20

$A = \{3, \{3\}, 1, 5, \{1,5\}, 6\}$, $B = \{3, \{1,5\}, \{7\}, 2, 7\}$

① $A - B = \{\{3\}, 1, 5, 6\}$

② $B - A = \{\{7\}, 2, 7\}$

③ $|P(A)| = 2^6 = 64$ ✓

$|A| = 6$

$|B| = 5$

④ T or F

ⓐ $\{3\} \in P(A)$ T

ⓑ $\{3\} \subseteq P(A)$ F

ⓒ $\{1,5,6\} \subseteq P(A)$ F

ⓓ $\{3,6\} \in P(A)$ T

ⓔ $\{\{1,5\}\} \subseteq P(A)$ T

ⓕ $\{\{3\}, 3, \{1,5\}\} \subseteq A$ T ✓

Name Sara Eltawil, ID 90097325

20

MTH 213, Discrete Math, Quiz 8

Ayman Badawi

QUESTION 1. 1) Let $a_n = a_{n-1} + 12a_{n-2}$, $a_1 = 2, a_2 = 4$. Find a general formula of a_n. Do not calculate C_1, C_2.

$a_n = c_1 \alpha^n + c_2 \alpha^n$

$\alpha^2 - \alpha - 12$

$\alpha = 4, -3$

6 ✓

$\boxed{a_n = 4^n c_1 - 3^n c_2}$

2) Let $a_n = a_{n-1} + 12a_{n-2} + 24$, $a_1 = 2, a_2 = 4$ (use 1, then find a general formula of a_n), calculate C_1 and C_2

$\boxed{a_n = 4^n c_1 - 3^n c_2 - 2}$

$a_n - a_{n-1} - 12a_{n-2} = 24$

$c - c - 12c = 24$

$-12c = 24$

$\boxed{c = -2}$

8

$a_1 = 2 \quad a_2 = 4$

$4c_1 - 3c_2 - 2 = 2$ (+2 +2)

$4^2 c_1 - 3^2 c_2 - 2 = 4$

$16c_1 + 9c_2 - 2 = 4$ (+2 +2)

$4c_1 - 3c_2 = 4$

$16c_1 + 9c_2 = 6$

$\boxed{c_1 = \frac{9}{14} \quad c_2 = \frac{-10}{21}}$ ✓

3) Let $= \{(1,1),(2,2),(3,3),(1,3),(3,2),(1,2)\}$. By staring, Is "=" an equivalence relation? if yes find all equivalence classes.

reflexive

NO, symmetry axiom fails

not an equivalence relation

3 ✓

4) Let <= {(1,1),(2,2),(3,3),(1,2),(3,2),(2,1)} Is " <= " a partial order? explain briefly

reflexive

3 ✓

NO, not partial order because of (1,2) and (2,1)

- anti symmetry axiom fails

Faculty information

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Name Israa , ID 900097435

$\frac{20}{20}$

Quiz V

Ayman Badawi

$\frac{14}{14}$

QUESTION 1. (14 points) T or F

(i) $\forall x \in Z, \exists y! \in Z$ such that $x + y = 0$ $\boxed{T}$ ✓ $y = -x$

(ii) $\forall x \in Z, \exists y! \in Z$ such that $x^2 - y^2 = 0$ $\boxed{F}$ ✓ $x^2 = y^2 \rightarrow y = \pm x$

F

(iii) If $\exists x \in R$ such that $x^2 + 1 = 0$, then $cos(x) = 2024$ $\boxed{T}$ ✓ $x^2 = -1 \rightarrow x = i$

(iv) Let $x \in Z$. Then $x^2 - x - 6 = 0$ if and only if $x + 22 = 25$ $\boxed{F}$ ✓

$x = 3, -2$

1) if $x^2 - x - 6 = 0$, then $x + 22 = 25$

x can be 3 or -2

F

2) if $x + 22 = 25$, then $x^2 - x - 6 = 0$

T

(v) $\exists! x \in N$ such that $\forall y \in R, yx - y = 0$ $\boxed{T}$ ✓

$x = 0, 1, 2, 3.$

$yx - y = 0 \rightarrow y(x-1) = 0$

$y = 0$ $x = 1$

(vi) $\forall y \in R, \exists! x \in N$ such that $yx - y = 0$ $\boxed{F}$ ✓

(vii) If $\exists x \in Z$ such that $x^2 - 4x = 0$, then $x^2 + 2 = 6$ $\boxed{F}$ ✓

$x = 4, 0$

QUESTION 2. 6 points Use the truth table and convince me that $S_1 + (S_2 \cdot S_3) \equiv (S_1 + S_2) \cdot (S_1 + S_3)$.

S_1	S_2	S_3	$S_2 \cdot S_3$	$S_1 + S_2 \cdot S_3$	$S_1 + S_2$	$S_1 + S_3$	$(S_1 + S_2)(S_1 + S_3)$
0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0
0	1	0	0	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1

same

$\frac{6}{6}$

Name Sara Elhawil, ID 900097325 — 20

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MTH 213, Discrete Math, Quiz 8

Ayman Badawi

QUESTION 1. 1) Let $a_n = a_{n-1} + 12a_{n-2}$, $a_1 = 2, a_2 = 4$. Find a general formula of a_n. Do not calculate C_1, C_2.

$a_n = c_1 \alpha^n + c_2 \alpha^n$ $\alpha^2 - \alpha - 12$

$\alpha = 4, -3$

6 ✓ $\boxed{a_n = 4^n c_1 - 3^n c_2}$

2) Let $a_n = a_{n-1} + 12a_{n-2} + 24$, $a_1 = 2, a_2 = 4$ (use 1, then find a general formula of a_n), calculate C_1 and C_2

$\boxed{a_n = 4^n c_1 - 3^n c_2 - 2}$

$a_n - a_{n-1} - 12a_{n-2} = 24$

$c - c - 12c = 24$

$-12c = 24$

$\boxed{c = -2}$

$a_1 = 2$ $a_2 = 4$

$4c_1 - 3c_2 - 2 = 2$ (+2 +2)

$4^2 c_1 - 3^2 c_2 - 2 = 4$

$16c_1 + 9c_2 - 2 = 4$ (+2 +2)

$4c_1 - 3c_2 = 4$

$16c_1 + 9c_2 = 6$

$\boxed{c_1 = \frac{9}{14} \quad c_2 = \frac{-10}{21}}$ ✓

8

3) Let = $\{(1,1),(2,2),(3,3),(1,3),(3,2),(1,2)\}$. By staring, Is "=" an equivalence relation? if yes find all equivalence classes.

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- anti symmetry axiom fails

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